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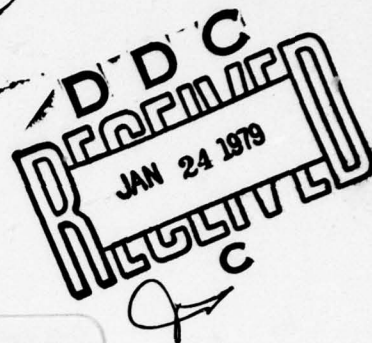
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COMPUTATIONAL EXPERIMENTS ON LARGE-SCALE OPTIMIZATION
WITH THE DECOMPOSITION PRINCIPLE

10 by
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12 48p.

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TECHNICAL REPORT SOL-78-33

11 December 1978

9 Technical rept.

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Research and reproduction of this report were partially supported by the National Science Foundation Grants MCS76-81259 A01; MCS76-20019 A01 and ENG77-06761 A01; the Office of Naval Research Contract N00014-75-C-0267 and the Department of Energy Contract EY-76-S-03-0326 PA #18.

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Abstract

↙ In this paper, ~~we trace out~~ ^{are traced out} the solution of large optimization problems for regional planning by a decomposition program that is based on the use of standard LP programs. The underlying method is the decomposition principle of Dantzig and Wolfe.

The concept is tested by solving an optimization problem with about 1250 columns and 900 rows. Furthermore, it is investigated to what extent the efficiency of the solution procedure can be influenced by an appropriate choice of starting solutions or specific matrix divisions.

↖

1. Introduction

The programming formulation of a large spatial equilibrium model of the agricultural sector of Germany, which is in preparation, results in a very large linear programming problem with a block-angular structure in the problem matrix. For solving it, existing standard computer programs for linear programming problems (standard LP programs) provide too little capacity.

In this paper, we trace out the solution of such problems by a decomposition program that is based on the use of Standard-LP-Programs. The underlying method is the decomposition principle of Dantzig and Wolfe [1]. The concept is tested by solving a similar but smaller problem with about 1250 columns and 900 rows. Furthermore, it is investigated to what extent the efficiency of the solution procedure can be influenced by the choice of starting solutions or specific matrix divisions.

Some of the results are of general interest for the solution of linear optimization problems with a similar matrix structure. They are based on experiments that have been partly reported in [5] and [6] and that are outlined in detail in section 4 of this paper.

2. Background information

Since 1970, a large spatial equilibrium model of the agricultural sector of Germany is in preparation. The first (completed) version of the model was designed as a linear optimization problem with about 8000 constraints. While this problem could still be solved by means of standard LP programs, the formulation of a more disaggregated version and/or the incorporation of other "Common Market" countries (suggestions which are

under consideration) would result in a model too large for the available programs (see appendix 1).

However, the special (block-angular) structure of the problem matrix that was due to the distinction of a limited number of "regions" as aggregated economic units of production and/or consumption¹⁾ would allow the solution of the problem by means of a computer program that is based on an appropriate decomposition method (see, for example, Geoffrion [2]).

However, despite the fact that decomposition methods are aimed at solving large-scale optimization problems, efficient programs for the solution of really large problems are not commonly available. Of the standard LP programs, only "LP 600/6000" and "OPTIMA" offer (as an alternative) a decomposition procedure that could be used for solving large-scale optimization problems. Unfortunately, these programs are designed for the use of specific computers only.

As the development of a specific decomposition program was not possible, it was suggested to base the solution of the optimization problem on a decomposition approach that allowed the use of one of the highly efficient standard LP programs. The approach that has been developed is, in principle, based on the decomposition method of Dantzig and Wolfe. It is outlined in more detail in the following chapter.

The efficiency of the concept has been tested in several experiments with a similar but smaller optimization problem. The organization of the experiments and the results of the computations are discussed in chapters 4 and 5 and in appendices 2, 3 and 4.

1) For a detailed discussion of such models see [7].

During the following discussion, we assume a familiarity with the decomposition principle. Its application requires the division of the optimization problem into "subproblems" and a "master problem".

The matrix of each subproblem is formed by one or several of the diagonal "submatrices" of the block-angular problem matrix.

The matrix of the master problem includes the common constraints of the problem matrix and, in addition, might also include one or several of the diagonal submatrices.

In an iterative solution procedure, the decomposition principle generates columns for the master problem from solutions of the subproblems and objective functions for the subproblems from solutions of the master problem.

3. Computational concept

In an optimization process based on the decomposition method of Dantzig and Wolfe [1], standard LP programs can be used for solving the subproblems and the master problem in each of the cycles of the iterative solution procedure. In this case, only computer programs have to be developed that transform the solutions of the subproblems and the master problems according to the requirements of the decomposition method.

Like most of the available standard LP programs for large problems (see appendix 1), the standard LP program that was available for the experiments (program XDLA of ICL) could not be used as a subprogram in computer programs which were written in a user-oriented programming language like ALGOL or FORTRAN.

The experiments were, therefore, based on a cycle of programs which included:

- 1) standard LP program for solving the subproblems;
- 2) FORTRAN program for generating columns for the master problem;
- 3) standard LP program for solving the master problem;
- 4) FORTRAN program for computing revised objective functions for the subproblems.

During the solution procedure, this cycle had to be activated repeatedly. Each repetition corresponded to one cycle of the solution procedure (i.e., one iteration of the decomposition process).

The optimization approach differed from the original presentation by Dantzig and Wolfe insofar as the following suggestions for shortening the solution procedure could be realized within the outlined concept:

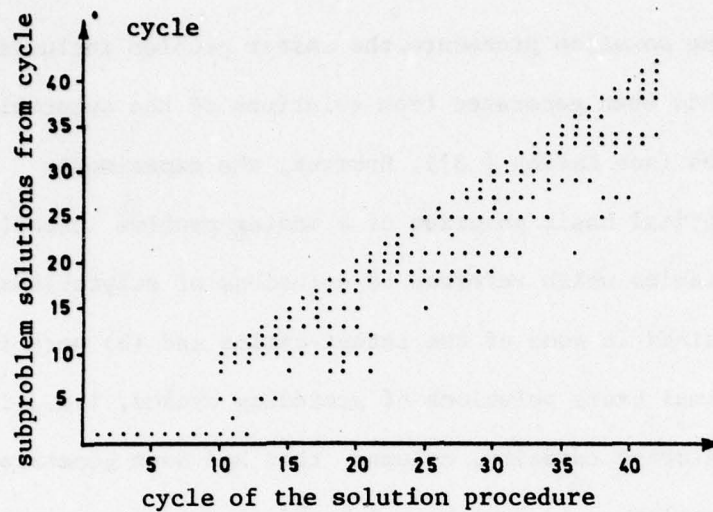


Exhibit 1: Solutions of subproblems that were included in the optimal basis solution of the master problem in different cycles (example: experiment I).

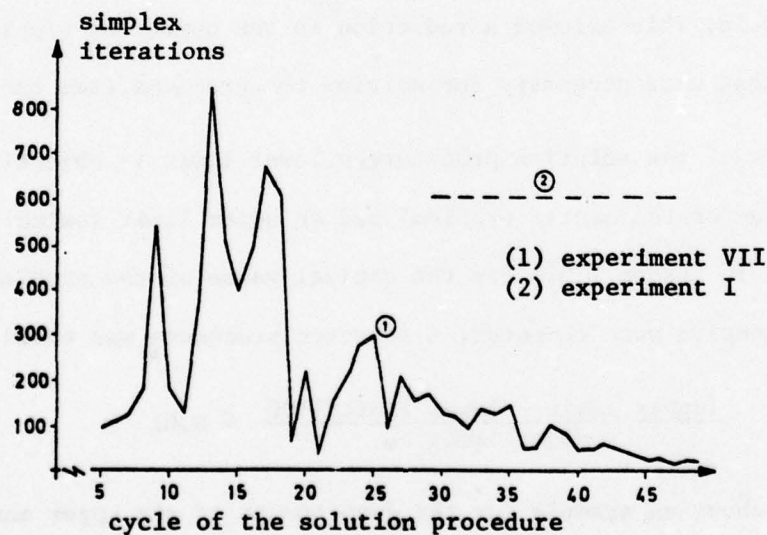


Exhibit 2: Number of simplex iterations that were necessary for solving the subproblems in different cycles. (In experiment VII, the optimizations were based on optimal basis solutions from preceding cycles; in experiment I, on the optimal basis solutions from the first cycle.)

- 1) In any cycle of the solution procedure, the master problem included all columns that had been generated from solutions of the subproblems in preceding cycles (see Lasdon [3]). However, the experiments showed that the optimal basis solution of a master problem usually included only variables which referred to solutions of subproblems that (a) were obtained in some of the latest cycles and (b) were included in the optimal basis solutions of preceding cycles, i.e., in cases of limited storage capacity, columns that had been generated during preceding cycles could have been deleted if the corresponding variables were no longer realized in the optimal basis solution of a master problem (see exhibit 1).
- 2) The optimization of subproblems and master problem in each cycle was based on the optimal basis solutions that had been obtained in the preceding cycle. This allowed a reduction in the number of simplex iterations that were necessary for solving the problems (see exhibit 2).
- 3) In each cycle of the solution procedure, a lower limit (= objective function value of the master problem) and an upper limit (calculated as described in Lasdon [3]) for the optimal value of the problem's objective function were computed. A solution procedure was ended when

$$\frac{(\text{upper limit} - \text{lower limit}) * 100}{\text{upper limit}} \leq 0.01 .$$

(Exhibit 3 shows an example for the development of the upper and lower limit for the objective function value of the optimal solution of the problem during the iterative solution procedure.)

objective func-
tion value
(million)

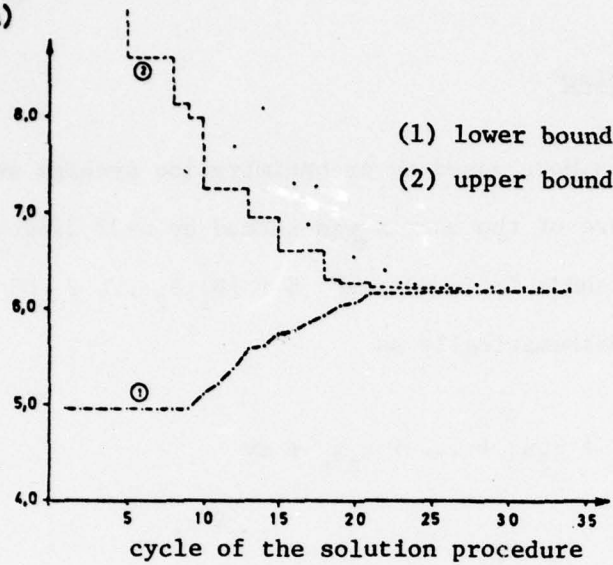


Exhibit 3: Upper and lower bound for the optimal value of the objective function in different cycles of the solution procedure (example: experiment I).

4. The experiments

4.1 Optimization problem

The experiments were based on an optimization problem where the block-angular structure of the matrix was formed by $n=32$ diagonal submatrices A_i ($i=1,\dots,n$) and a "main matrix" $H = [B_1 \ B_2 \ \dots \ B_n \ D]$ and which could be formulated mathematically as

$$\begin{aligned}
 \max z &= c_1 x_1 + c_2 x_2 + \dots + c_n x_n + dy \\
 \text{s.t.} \quad & A_1 x_1 \leq a_1 \\
 & \quad A_2 x_2 \leq a_2 \\
 & \quad \quad \vdots \\
 & \quad \quad A_n x_n \leq a_n \\
 & B_1 x_1 + B_2 x_2 + \dots + B_n x_n + Dy = b \\
 & x_i, y \geq 0 \quad (i=1,\dots,n)
 \end{aligned}$$

with x_i ($i=1,\dots,n$) and y representing vectors of decision variables.

In the following, we will refer to the constraints that are connected with the main matrix H as "common" constraints. The size and density of the different matrices are outlined in exhibit 4.

Matrix size (rows,columns)	(907,1265)
Density	0.71
Number of submatrices	32
Size (rows,columns)	(28,39)
Density	18.58
Size of the main matrix	(11,1265)
Density	7.73

Exhibit 4

4.2 Organization of the experiments

The experiments were aimed at testing the efficiency of the computational concept and investigating ways of improving the convergence of the iterative solution process by

- a) an appropriate choice of the feasible starting solution that has to be formulated at the beginning of the solution process (see experiments I, II, III and VII),
- b) the incorporation of submatrices into the master problem (see experiments I, IV and V) and
- c) the variation of the size of distinguished subproblems (see experiments VI, VII and X).

The organization of the different experiments is outlined in detail in exhibit 5 .

The distinguished starting solutions A, B, C and D (see column 2 of the exhibit) were feasible solutions of the optimization problem that differed

- a) in the values of their objective functions (except solutions A and D),
- b) in the values of the dual variables that corresponded to the common constraints (except solutions A and B) and
- c) in the values of the primal variables.

The number of constraints in the master problem (see column 6) refers to the common constraints of the optimization problem, the additional constraints necessary for forming convex combinations of subproblem proposals and (in experiment IV and V) constraints that are due to the incorporation of submatrices into the master problem.

Experiment	Identification of starting solution	Deviation of the objective function value*)	Number of subproblems	Matrix size of subproblem (m,n)	Number of constraints in the master problem	Main aspect of investigation
I	A	-20 %	4	224,312	11 + 4	Different starting solutions
II	B	-40 %	4	224,312	11 + 4	
III	C	-50 %	4	224,312	11 + 4	
VII	D	-20 %	4	224,312	11 + 4	
VIII	A+B+C+D	-16 %	4	224,312	11 + 4	
I	A	-20 %	4	224,312	11 + 4	Incorporation of submatrices into the master problem
IV	A	-16 %	3	224,312	11 + 224 + 3	
V	A	-14 %	2	224,312	11 + 448 + 2	
VI	D	-20 %	8	112,156	11 + 8	Different size of subproblems
VII	D	-20 %	4	224,312	11 + 4	
X	D	-20 %	1	896,1248	11 + 1	

*) Deviation of the master problem's optimal objective function value from the optimal value of z at the beginning of the iterative solution procedure.

Exhibit 5 : Organization of the basic experiments.

In addition to these basic experiments some computations were performed in which

- a) the objective function of the optimization problem was changed in one of the experiments after an optimal solution with regard to the original objective function had been reached (see experiment IX in chapter 4.36) and
- b) differences in common constraints that had been observed in the basic experiments were examined with regard to their influence on the optimization process (see experiments XI, XII, XIII and XIV in appendix 2).

4.3 Discussion of results

We will discuss the results of the experiments with regard to the usefulness of an approach in terms of its effects on

- a) the computation time and
- b) the generation of "good" suboptimal solutions at an early stage of the iterative solution procedure.

In the following context, a suboptimal solution is considered to be "good" if it is a sufficiently good approach to the optimal solution, i.e., an approach that would allow the termination of the iterative solution procedure if some deviation between the computed solution and the optimal solution is considered to be acceptable.

4.31 Relevance of results

The results that are of relevance for an experiment's computation time are outlined in exhibits 6 and 7 .

Experiment	Number of cycles	Number of subproblems generated solutions (total)	Number of simplex iterations in the subproblems (average)		Number of simplex iterations in the master problem		Calculations of the type $a=bt+cd$ (in 1000)	
			per subproblem	per cycle	per cycle	total	per cycle (average)	total
I	42	168	150	600	5	210 ¹⁾	28.7	1205
II	40	160	48	192	5	200	28.7	1148
III	42	168	52	210	5	210	28.7	1205
VII	49	196	57	228	5	245	28.7	1406
VIII	40	172	50	201	5	200	28.7	1148
I	42	168	150	600	5	210 ¹⁾	28.7	1205
IV	13	39	59	178	63	819	21.5	280
V	10	20	59	118	175	1753	14.4	144
VI	35	280	29	233	7	245	28.7	1005
VII	49	196	57	228	5	245	28.7	1406
X	103	103	138	138	2	206	28.7	2956
Number of simplex iterations necessary for solving the problem without decomposition: <div> a) with starting solution A: 1729 b) with starting solution D: 1582 </div>								

1) The optimization was not based on the optimal basis solution of a preceding cycle.

Exhibit 6 : Some results of the experiments.

Experiment	Average core time (min:sec)		Average transfer time (min:sec)		Total time (hrs:min)	
	per sub- problem	per master problem	per sub- problem	per master problem	core time	core+trans- fer time
I	3:00	0.09	5:25	4:30	-	- *)
II	1:11	0.09	5:25	4:30	4:47	28:03
III	1:18	0.09	5:25	4:30	5:21	29:47
VII	1:22	0.09	5:25	4:30	6:27	34:57
VIII	1:11	0.09	5:25	4:30	4:47	28:03
I	3:00	0.09	5:25	4:30	-	- *)
IV	1:27	2:04	5:25	6:40	1:45	8:12
V	1:24	12:06	5:25	9:10	2:40	6:50
VI	0:27	0.09	3:45	6:30	3:31	31:48
VII	1:22	0.09	5:25	4:30	6:27	34:57
X	11:35	0.09	21:20	2:50	24:05	78:52
Solving the problem without decomposition:					4:14	
a) with starting solution A					4:00	
b) with starting solution D						

*) The optimizations were not based on the optimal basis solution of a preceding cycle.

Exhibit 7 : Some results of the experiments.

In exhibit 7, the terms "core time" and "transfer time" are used. In this context, the term "core time" refers to the time that was necessary for performing the simplex iterations in the subproblems and the master problems and for performing the calculations of type $a=b+c*d$, the calculation of indices, etc. in the FORTRAN programs. An identification of the relevant CPU time was not possible. The term "transfer time" refers mainly to the time that was necessary for

- a) exchanging data between LP programs and FORTRAN programs (with discs as temporary storage units) and
- b) the transfer of subproblems and master problems from the central processor unit to storage units (discs) and reverse.

At any time, there was only one optimization problem (a subproblem or a master problem) in the central processor unit for performing computations.

The specified average core and transfer times are closely related to the used computer facilities and the organization of the programming cycle whereas the other figures in the exhibits are related to the optimization problem that had to be solved.

It should be noted that the percentage of computation time that could be specified as core time was relatively low in all experiments (between 11% and 39%), i.e., an efficient organization of the storage facilities is one of the most important possibilities for increasing the efficiency of the optimization process. This is especially true with regard to the LP programs which were responsible for most of the needed computation time (usually for more than 75%).

Of a more general interest are the variations of results within the distinguished groups of experiments. Their analysis can provide information which could be of interest for the optimization of problems with a similar structure in the problem matrix. The same is true for a comparative evaluation of the suboptimal solutions that were generated during the iterative solution procedures.

The more important observations and conclusions are discussed in detail in the following sections.

4.32 Different starting solutions

A comparison of different starting solutions was based

- a) on differences in their objective function value,
- b) on deviations of the values of the dual variables that refer to the common constraints from their values in the optimal solution and
- c) on deviations of the values of primal variables from their values in the optimal solution.

A comparison of the experiments I, II, III, and VII showed that it is not possible to judge the usefulness of a specific starting solution merely on the basis of one of these criteria, not to mention the problem of determining at least some rough estimates for the optimal values of the objective function or the primal and dual variables.

For example, the experiments I, II and III could be finished in an approximately identical number of cycles despite the fact that the objective function values of the starting solutions differed significantly (see exhibit 6).

Furthermore, differences in the objective function values of different starting solutions were of no relevance for the development of these values during the solution procedure (see exhibits 8 and 9).

The same conclusion could be drawn with regard to the values of the dual variables that correspond to the common constraints and with regard to the values of the primal variables which were examined by distinguishing

- a) variables with coefficients in the main matrix only,
- b) variables with coefficients in the submatrices only and
- c) other variables.

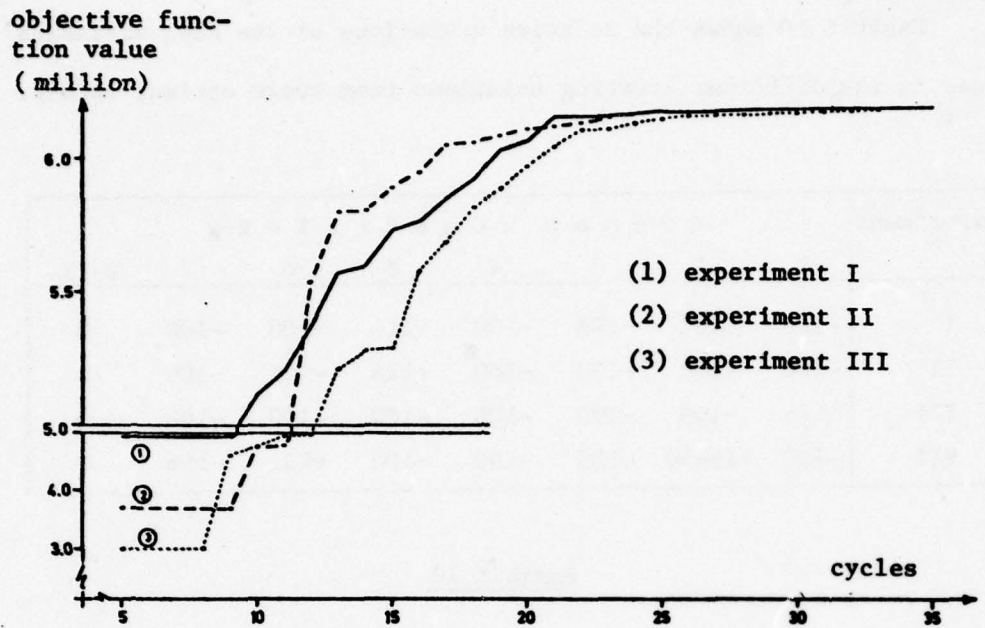


Exhibit 8: Development of the objective function values during the experiments I, II and III.

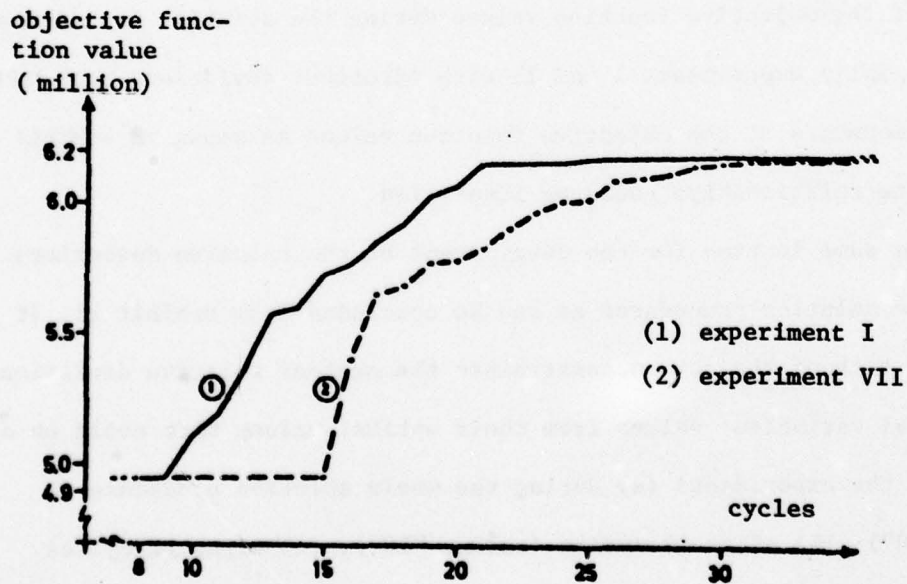


Exhibit 9: Development of the objective function values during the experiments I and VII.

Exhibit 10 shows the relative deviations of the dual variables' values in the different starting solutions from their optimal values.

Experiment	Common constraints							
	1	2	3	4	5	6	7	8-11
I	-100	-100	-100	-100	+114	-100	-100	0
II	-100	-100	-100	-100	+114	-100	-100	0
III	+676	-100	-100	-100	-100	-100	-100	0
VII	-100	+15450	-100	-100	-100	+431	-196	0

Exhibit 10

Comparing them with the number of cycles that were necessary for performing the experiments (see especially experiments I and III with different deviations but identical numbers of cycles) and the development of the objective function values during the solution procedures (see especially experiments I and II with identical deviations but different developments of the objective function values as shown in exhibit 8) no definite relationships could be identified.

The same is true for the development of the relative deviations during the solution procedures as can be concluded from exhibit 11. It shows for each of the common constraints the maximal relative deviations of the dual variables' values from their optimal values that could be observed in the experiments (a) during the whole solution procedure (column "0"), (b) after 10 cycles (column "10"), (c) after 20 cycles (column "20") and (d) after 30 cycles (column "30").

Experiment	Common constraint															
	<u>1</u>				<u>2</u>				<u>3</u>				<u>4</u>			
	0	10	20	30	0	10	20	30	0	10	20	30	0	10	20	30
I	167	54	39	7	100	54	29	21	100	24	11	5	100	66	19	11
II	295	58	16	5	120	80	29	25	100	13	13	7	100	76	21	1
III	676	61	24	7	165	165	73	29	100	12	8	4	100	100	17	2
VII	199	199	34	8	15450/2329/29/29				100	63	10	10	100	100	45	3
Common constraint																
	<u>5</u>				<u>6</u>				<u>7</u>				8-11			
	0	10	20	30	0	10	20	30	0	10	20	30				
I	114	100	33	18	690	142	79	11	196	196	45	12	0			
II	114	100	76	9	444	109	18	18	196	196	78	13	0			
III	114	100	45	45	620	228	77	25	196	196	38	7	0			
IV	114	114	100	21	690	690	132	38	196	196	99	31	0			

Exhibit 11

objective function value
(million)

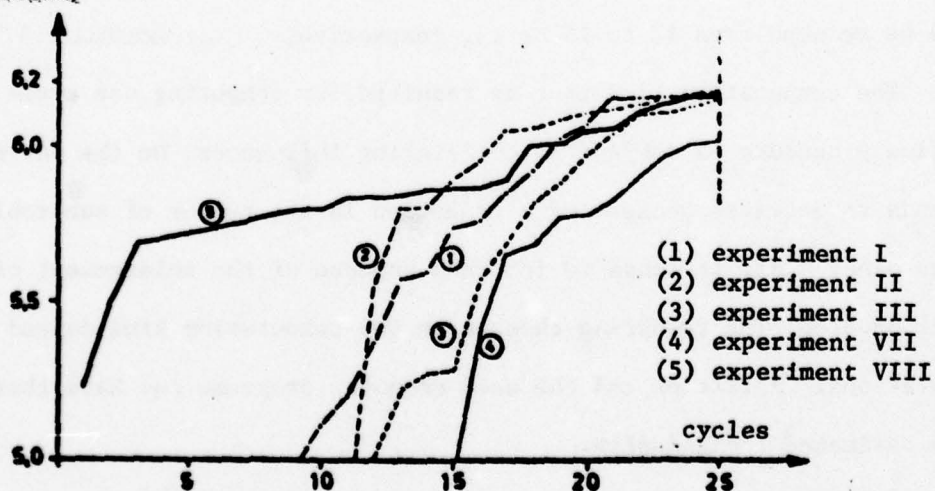


Exhibit 12: Development of the objective function value during experiment VIII (for a comparison see experiments I, II, III and VII).

However, the determination of a "useful" starting solution could be simplified by specifying not a single solution but several ones, each of which might be less useful or even not feasible but which could be combined to a more useful "starting solution" in the first master problem of the solution procedure (see experiment VIII and exhibit 12).

4.33 Incorporation of submatrices into the master problem

The incorporation of submatrices into the master problem (while reducing the number of subproblems) results

- a) in a higher flexibility of the master problem and
- b) usually in a change in the time that is required for computing one cycle of the solution procedure.

Because of the master problem's higher flexibility in the experiments IV and V compared to experiment I it was possible to reach in the first iteration objective function values that differed from the optimal value not more than 16 % or 14 %, respectively. In addition, the number of cycles could be reduced from 42 to 15 or 13, respectively. (See exhibit 13.)

The computation time that is required for computing one cycle of the solution procedure is subject to conflicting influences. On the one side, it tends to decrease because of a reduction in the number of subproblems, on the other side, it tends to increase because of the enlargement of the master problem. The resulting changes in the computation time depend on the computational facilities and the used computer programs and have, therefore, to be estimated individually.

For example, a comparison of experiments IV and V showed that in this case the incorporation of an additional submatrix into the master problem

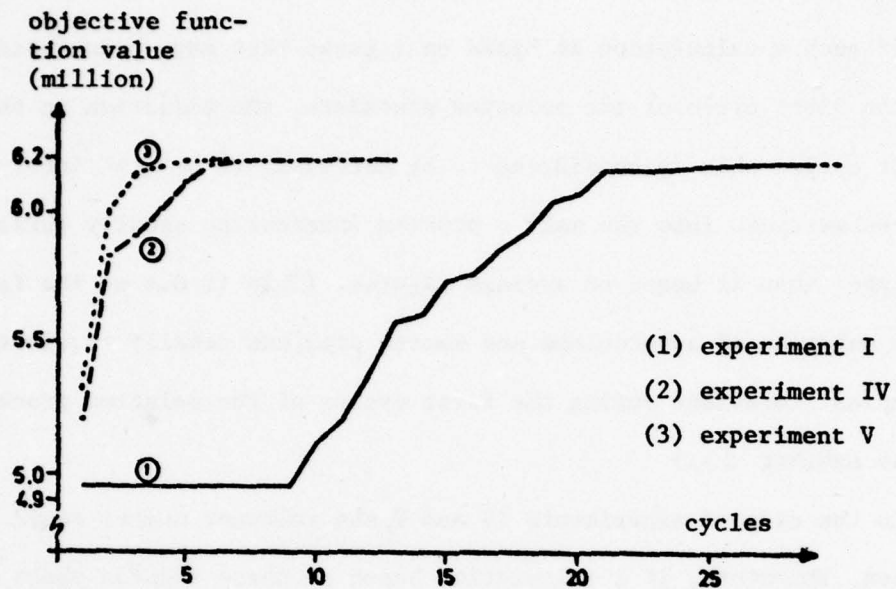


Exhibit 13: Development of the objective function values during the experiments I, IV and V.

would be of interest if the number of cycles could be reduced by (theoretically) 0.1.

If such a calculation is based on figures that have been obtained during the first cycle of the solution procedure, the reduction in the number of cycles that is considered to be necessary to make an incorporation of submatrices into the master problem interesting usually turns out to be higher than if based on average figures. (This is due to the fact that the solution of subproblems and master problems usually requires more simplex iterations during the first cycles of the solution procedure (see also exhibit 2).)

In the case of experiments IV and V, the relevant number would be 0.5 cycles. Therefore, if a calculation based on these figures shows an incorporation of submatrices into the master problem to be efficient, it can usually be implied that it will be efficient with regard to the (still unknown) average figures as well.

4.34 Variations in the number and size of subproblems

Changes in the number and size of distinguished subproblems influence (a) the number of cycles that are necessary for solving an optimization problem and (b) the time required for performing a single cycle within the solution procedure.

Theoretically, the number of necessary cycles can be minimized by distinguishing as many subproblems as possible (see, for example, Lasdon [3]).

As an example, the number of cycles that were necessary for performing the experiments X, VII and VI could be reduced from 103 to 49 and 35.

In addition, by distinguishing more but smaller subproblems it was possible to reduce the average time that was necessary for performing the simplex iterations in a cycle of the procedure. This was due to the fact that the computation of a simplex iteration takes less time in smaller problems and that this reduction in computation time was not compensated in the experiments by an observed slight increase in the number of simplex iterations.

However, due to the use of standard LP programs, the increase in the number of distinguished subproblems is only useful within a certain range. The activation of these programs requires a basic amount of time which is independent of the size of an optimization problem. Because of this, the average time for computing a cycle in experiment VI was about 500 seconds longer than in experiment VII, while the transition from experiment X to experiment VII still caused a reduction in computation time of about 250 seconds.

The extent to which an increase in the number of distinguished subproblems remains useful can only be estimated individually. If such an estimation is based on figures that have been obtained during one of the first cycles of the solution procedure, it has to be taken into account that the solving of subproblems usually requires more simplex iterations at the beginning than towards the end of the solution procedure, i.e., the possible savings in computation time are usually lower than indicated during the first cycles of the procedure.

4.35 General remarks about the values of primal and dual variables

In situations where the solution process has to be ended before an optimal solution has been reached, the values of the primal and dual variables and their deviations from the optimal values might be of interest.

In the experiments, the values of the primal variables usually varied only within a relatively small range within the feasible range during the iterative process (disregarding the very first cycles). However, the deviations from their optimal values could still be relatively high (with regard to the value of the objective function) in almost optimal solutions (see exhibit 14). There were no significant differences between variables with

- a) coefficients only in the common constraints,
- b) coefficients only in the submatrices and
- c) other variables.

The values of the dual variables (that corresponded to the common constraints) that had been observed during the iterative solution processes are outlined in appendix 4. They varied significantly around their optimal values and usually showed some kind of convergence properties, as demonstrated in exhibit 15 (see also exhibit 11).

4.36 Postoptimal calculations

For the examination of the effects of changes in the objective function or the capacity vector of an optimization problem on the computed optimal solution, the iterative solution procedure has to be restarted.

However, the columns of the last cycle's master problem can be included in the master problem of the restarted process if changes in the

number of
units

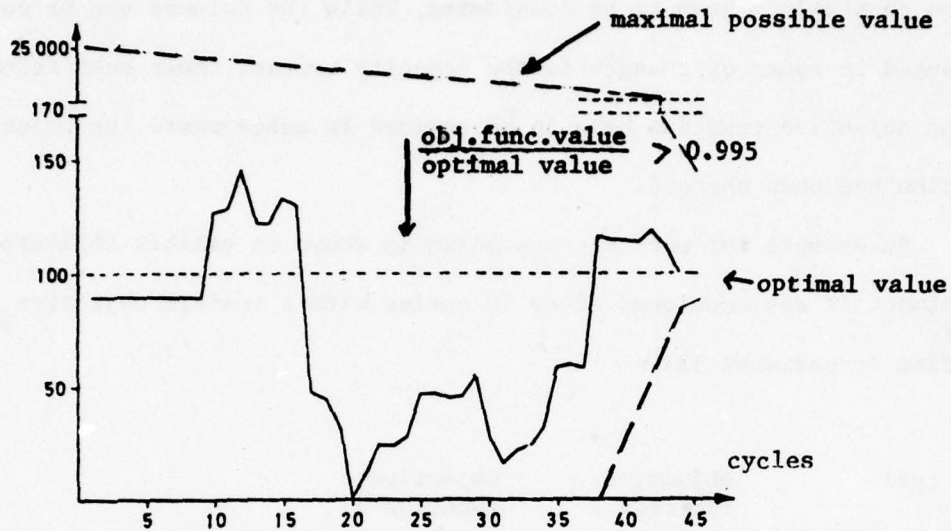


Exhibit 14: Changes in the value of a primal variable during experiment I.

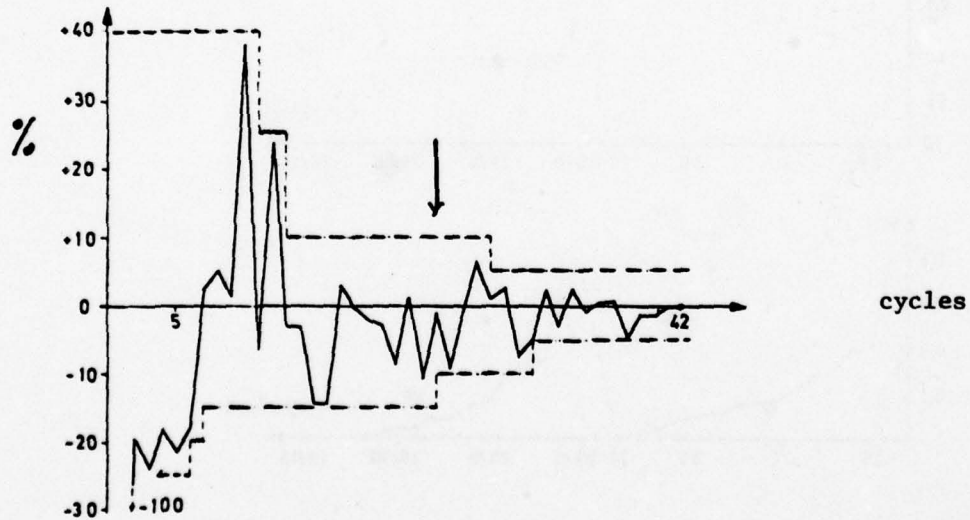


Exhibit 15: Changes in the value of a dual variable (that corresponded to one of the common constraints) during experiment I (deviations from the optimal value in %).

objective function or the part of the capacity vector that refers to the common constraints have to be considered. While the columns can be used unchanged in cases of changes in the capacity vector, their coefficients in the objective function have to be revised in cases where the objective function had been changed.

An example for such a computation is shown in exhibit 16, where experiment II was continued after 39 cycles with a revised objective function (experiment IX).

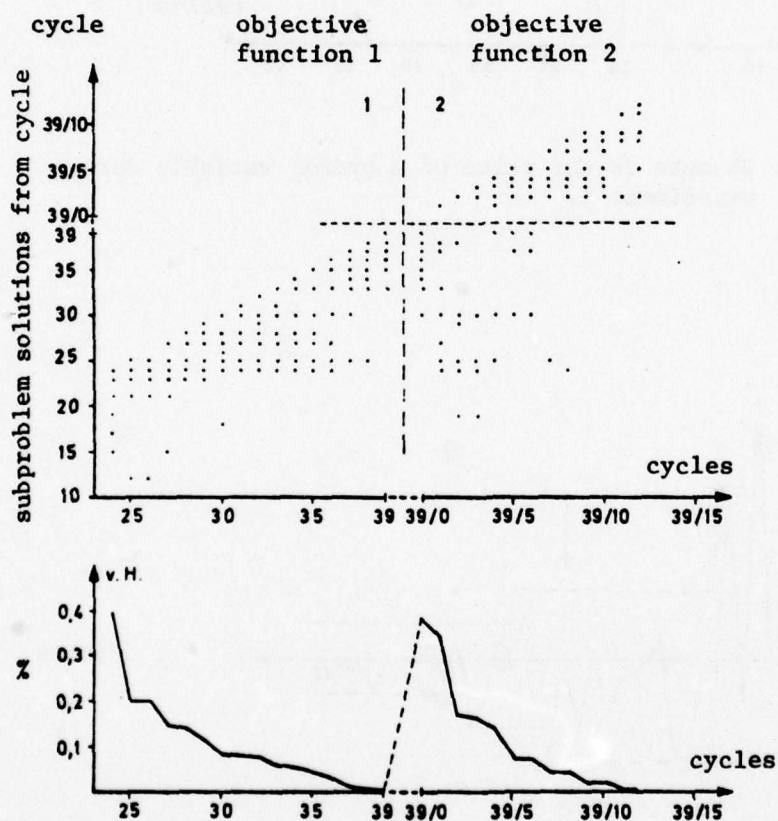


Exhibit 16: Solutions of subproblems that were included in the optimal basis solution of the master problem and the relative deviations of the objective functions' values from their optimal values when objective function 1 was replaced by objective function 2 (experiment IX).

5. Conclusion

The solution of large optimization problems with a decomposition program that is based on efficient standard LP programs is usually no alternative to a simultaneous optimization, i.e. an optimization without decomposition by means of a standard LP program. The realization of the computational concept requires additional efforts for establishing the programming cycle, organizing the data etc., which reduces its efficiency.

However, for the solution of optimization problems that are too large for the available computational facilities, the proposed procedure might be efficient.

As the experiments showed, an appropriate identification of sub-problems could allow the reduction of core time that is necessary for solving an optimization problem to an amount similar to the amount of time that is necessary for a simultaneous optimization (see, for example, exhibit 5).

Appendix 1: Standard LP programs for solving large-scale optimization problems

The size of optimization problems that can be solved by means of standard LP programs is limited by

- a) the capacity of the available computers and
- b) the efficiency of the standard LP programs.

The crucial figure in this context is the number of constraints that have been formulated in the problem (see exhibit 17).

Name of the program (Company)	Rows (m)	Columns (n)
FMPS (UNIVAC)	8 192	unlimited
ILONA (UNIVAC)	$m+n = 9\ 500$	
LP 4004 (SIEMENS)*	6 000	32 767
LP 600/6000 (HONEYWELL BULL)*	4 095	262 000
MARIE CLAIRE (UNIVAC)	12 000	unlimited
MPSX (IBM)	16 383	unlimited
OPHELIE II (CONTROL DATA)*	10 000	unlimited
OPTIMA (CONTROL DATA)*	4 094	unlimited
XDLA (ICL)	25 000**	100 000
* Information from 1972, others 1973 (XDLA 1974)		
** Problems with m=2500 reported		

Exhibit 17

For problems with a block-angular structure in the matrix the standard LP programs "LP 600/6000" and "Optima" offer an alternative solution procedure which is based on a decomposition method developed by Orchard-Hays [4] and which is designed for solving problems with up to 50,000 constraints. However, practical experiences with this procedure seem to be very scarce.¹⁾

1) Orchard-Hays in private correspondence.

Appendix 2: Additional experiments with different common constraints

In the experiments I - X, it turned out that some of the dual variables that corresponded to the common constraints always reached their optimal values in the first cycle of the solution procedure while the remaining dual variables changed their values iteratively. The common constraints that corresponded to the first group of dual variables were numbered 8 - 11, the remaining common constraints 1 - 7.

For getting more information about the influence of different kinds of common constraints on the solution procedure, experiments XI, XII, XIII and XIV were outlined, basically as described for experiment X but with the following specifications:

- 1) In experiments XI and XII, the master problems included only one of the common constraints whereas the remaining common constraints were added to the subproblems. In experiment XI we considered explicitly constraint 11, in experiment XII constraint 1.
- 2) In experiments XIII and XIV, the master problems included only one of the distinguished groups of common constraints whereas the remaining group of common constraints was added to the subproblems. In experiment XIII we considered explicitly the common constraints 8-11 in the master problems, in experiment XIV the common constraints 1-7.

The results of the computations are outlined in exhibits 18 to 19.

The most noteworthy results seem to be:

- a) the similarity in the experiments XI and XII despite the fact that different kinds of common constraints were considered,
- b) the solution procedure of experiment XIII.

Cycle	Lower bound	Upper bound computed in cycle	"Lowest" computed upper bound
-------	-------------	-------------------------------	-------------------------------

Experiment XI

1	4 955 266		
2	.	6 268 201	6 268 201
3	6 140 388	17 117 741	.
4	6 181 931	6 481 359	.
5	6 193 908	6 248 256	6 248 256
6	6 194 023	6 194 531	6 194 531
7	6 194 083	6 194 097	6 194 097

Experiment XII

1	4 955 266		
2	.	8 559 708	8 559 708
3	.	6 460 372	6 460 372
4	6 038 806	6 279 672	6 279 672
5	6 143 447	6 234 462	6 234 462
6	6 187 820	6 195 371	6 195 371
7	6 191 760	6 199 105	.
8	6 193 735	6 194 327	6 194 327
9	6 194 013	6 194 344	.
10	6 194 082	6 194 083	6 194 083

Experiment XIII

1	4 955 266		
2	6 194 083	6 194 083	6 194 083

Experiment XIV

1	4 955 266		
2	.	108 019 791	108 019 791
3	.	12 857 900	12 857 900
4	.	15 858 291	.
5	.	13 687 413	.
6	.	16 955 389	.
7	.	8 986 892	8 986 892
8	.	12 328 003	.
9	.	9 698 789	.
10	.	9 230 287	.
11	.	8 923 979	8 923 979
12	.	9 760 913	.
13	.	8 559 648	8 559 648
14	.	7 945 981	7 945 981
15	.	10 879 345	.
16	.	12 621 243	.
17	.	8 623 734	.
18	.	8 204 922	.
19	5 335 049	7 142 742	7 142 742
20	5 425 695	14 211 884	.
21	5 674 791	9 989 577	.
22	5 678 287	7 737 021	.

experiment ended

Exhibit 18: Development of the lower and upper bound for the optimal value of the objective function in experiments XI, XII, XIII and XIV. (Optimal value: 6,194,083).

Cycle	1	2	3	4	5	6	7	8	9	10
Experiment XI: (common constraint 11)	0	-1073.6	-101.0	-68.2	-58.7	-58.6	-58.6	-	-	-
Experiment XII: (common constraint 1)	0	100.1	311.3	159.3	225.3	193.1	209.9	218.3	214.4	215.2
Experiment XIII: (common con- straints 8-11)	optimal values in first cycle (remained unchanged)									

Exhibit 19: Values of the dual variables that corresponded to the common constraints in experiments XI (common constraint 11), experiment XII (common constraint 1) and experiment XIII (common constraints 8-11) during the solution procedure.

Appendix 3: Bounds for the optimal objective function values in the basic experiments

The exhibits in this section show for the basic experiments the development of

- a) a lower bound for the optimal objective function value which equals the objective function value of a cycle's master problem (in the exhibits referred to as "lower bound"),
- b) an upper bound for the optimal objective function value which was computed in each cycle in the way suggested by Lasdon [3] (in the exhibits referred to as "upper bound in cycle") and
- c) the lowest upper bound for the optimal objective function value that had been computed in preceding cycles (in the exhibits referred to as "upper bound").

Cycle Lower bound Upper bound
in cycle

1	4	955	266	212	517	341
2	.	.	.	12	993	362
3	.	.	.	14	345	848
4	.	.	.	8	626	938
5	.	.	.	15	122	051
6	.	.	.	9	356	478
7	.	.	.	8	142	341
8	.	.	.	7	985	506
9	.	.	.	7	241	441
10	5	113	444	7	241	441
11	5	201	369	11	496	234
12	5	373	734	7	686	574
13	5	576	173	6	934	658
14	5	600	763	8	088	533
15	5	734	756	6	603	113
16	5	765	017	7	299	943
17	5	853	308	6	872	263
18	5	920	986	6	297	866
19	5	031	620	6	274	664
20	6	068	026	6	548	404
21	6	155	005	6	221	113
22	6	158	179	6	400	293
23	6	158	278	6	270	954
24	6	164	156	6	245	333
25	6	175	879	6	205	714
26	6	176	095	6	245	292
27	6	179	075	6	206	331
28	6	179	304	6	252	788
29	6	180	533	6	217	509
30	6	186	087	6	204	441
31	6	186	576	6	219	973
32	6	188	631	6	202	570
33	6	190	127	6	199	092
34	6	190	849	6	198	342
35	6	191	401	6	209	804
36	6	191	678	6	199	230
37	6	192	020	6	196	573
38	6	193	424	6	195	243
39	6	193	735	6	197	707
40	6	193	785	6	194	768
Opt.	6	193	822	6	194	673
42	6	193	879	6	194	324
Opt.	6	194	083	6	194	324

Experiment I

Cycle Lower bound Upper bound
in cycle

1	3	716	450	212	517	341
2	.	.	.	12	886	689
3	.	.	.	14	259	235
4	.	.	.	11	605	810
5	.	.	.	8	665	219
6	.	.	.	16	196	403
7	.	.	.	8	314	422
8	.	.	.	9	784	374
9	.	.	.	8	022	513
10	4	750	103	8	022	513
11	4	785	851	8	258	896
12	5	540	575	6	741	136
13	5	803	604	7	152	522
14	5	807	867	6	936	295
15	5	823	770	6	696	114
16	5	952	703	6	481	977
17	5	053	890	6	538	358
18	6	060	872	6	455	696
19	6	087	795	6	269	133
20	6	110	669	6	245	987
21	6	120	900	6	245	849
22	6	133	251	6	410	246
23	6	152	305	6	242	482
24	6	169	005	6	211	482
25	6	181	561	6	234	677
26	6	181	561	6	310	339
27	6	184	824	6	221	685
28	6	185	401	6	214	304
29	6	186	883	6	214	457
30	6	188	790	6	198	919
31	6	188	941	6	248	248
32	6	189	139	6	205	687
33	6	190	104	6	196	542
34	6	190	460	6	214	937
35	6	191	343	6	197	037
36	6	192	019	6	198	797
37	6	193	250	6	195	664
38	6	193	491	6	197	723
39	6	193	683	6	194	589
40	6	194	027	6	194	457
Opt.	6	194	083	6	194	457

Experiment II

Cycle	Lower bound	Upper bound in cycle	Upper bound
1	5	202	708
2	5	853	010
3	5	902	600
4	5	014	362
5	6	111	826
6	6	176	351
7	6	184	451
8	6	186	787
9	6	188	251
10	6	190	315
11	6	191	645
12	6	193	618
13	6	193	812
14	6	193	930
15	6	194	047
Opt.	6	194	083

Experiment IV

Cycle	Lower bound	Upper bound in cycle	Upper bound
1	5	327	386
2	5	991	934
3	6	135	900
4	6	163	200
5	6	181	622
6	6	192	884
7	6	192	896
8	6	193	470
9	6	193	653
10	6	194	048
11	6	194	068
12	6	194	082.6432
13	6	194	083.0314
14	6	194	083.0314
Opt.	6	194	083.0314

Experiment V

Cycle	Lower bound	Upper bound in cycle	Upper bound
1	2	997	090
2	.	.	.
3	.	.	.
4	.	.	.
5	.	.	.
6	.	.	.
7	.	.	.
8	4	616	759
9	4	713	943
10	.	.	.
11	4	940	078
12	4	988	826
13	5	212	468
14	5	274	927
15	5	290	075
16	5	579	064
17	5	685	244
18	5	819	794
19	5	887	304
20	5	979	091
21	6	043	182
22	6	106	552
23	6	110	099
24	6	130	099
25	6	151	247
26	6	158	898
27	6	159	593
28	6	160	541
29	6	168	317
30	6	174	044
31	6	176	735
32	6	184	924
33	6	187	831
34	6	193	143
35	6	193	173
36	6	193	298
37	6	193	298
38	6	193	335
39	6	193	335
40	6	193	390
41	6	193	508
42	6	193	890
43	6	193	921
44	6	194	012
Opt.	6	194	083

Experiment III

Cycle Lower bound Upper bound
in cycle

1	4	955	266	275	169	947	275	169	947
2				85	975	784	85	975	784
3				72	415	707	72	415	707
4				18	910	950	18	910	950
5				15	495	504	15	495	504
6				12	791	332	12	791	332
7				11	505	378	11	505	378
8				15	003	472	15	003	472
9	5	003	687	7	905	338	7	905	338
10	5	348	966	7	905	338	7	905	338
11	5	771	999	7	995	469	7	995	469
12	5	807	214	11	282	227	11	282	227
13	5	834	155	7	791	450	7	791	450
14	5	834	392	7	928	540	7	928	540
15	5	856	340	6	947	648	6	947	648
16	5	862	133	7	053	991	7	053	991
17	5	938	393	6	494	368	6	494	368
18	5	967	046	7	482	284	7	482	284
19	6	017	127	6	446	273	6	446	273
20	6	019	396	6	466		6	466	
21	6	027	602	6	379	788	6	379	788
22	6	044	830	6	518	158	6	518	158
23	6	106	586	6	276	437	6	276	437
24	6	165	907	6	215	197	6	215	197
25	6	172	528	6	246	574	6	246	574
26	6	173	751	6	270	626	6	270	626
27	6	178	467	6	234	426	6	234	426
28	6	184	612	6	220	223	6	220	223
29	6	188	013	6	203	378	6	203	378
30	6	188	216	6	210	289	6	210	289
31	6	191	121	6	198	954	6	198	954
32	6	192	560	6	199	019	6	199	019
33	6	192	847	6	195	925	6	195	925
34	6	193	776	6	194	559	6	194	559
35	6	193	984	6	194	362	6	194	362
36	6	194	041	6	194	320	6	194	320
Opt.	6	194	083						

Experiment VI

Cycle Lower bound Upper bound
in cycle

1	5	227	982	228	715	619	228	715	619
2	5	501	727	85	691	850	85	691	850
3	5	690	727	13	372	397	13	372	397
4	5	704	102	11	580	588	11	580	588
5	5	724	149	8	918	861	8	918	861
6	5	740	149	8	707	237	8	707	237
7	5	756	676	6	850	009	6	850	009
8	5	786	609	8	560	608	8	560	608
9	5	799	482	9	039	583	9	039	583
10	5	800	453						
11	5	838	345	7	163	590	7	163	590
12	5	840	019	8	393	635	8	393	635
13	5	846	760	6	666	933	6	666	933
14	5	864	841	7	071	689	7	071	689
15	5	867	116	6	522	057	6	522	057
16	5	868	711	6	352	371	6	352	371
17	5	896	414	6	580	268	6	580	268
18	6	008	318	8	192	655	8	192	655
19	6	014	415	6	538	546	6	538	546
20	6	056	013						
21	6	061	601	6	391	291	6	391	291
22	6	116	450	6	264	739	6	264	739
23	6	139	180	6	222	478	6	222	478
24	6	157	722	6	243	964	6	243	964
25	6	157	943	6	300	824	6	300	824
26	6	164	836	6	238	665	6	238	665
27	6	172	897	6	211	148	6	211	148
28	6	183	812	6	199	922	6	199	922
29	6	183	812	6	236	700	6	236	700
30	6	186	051	6	217	184	6	217	184
31	6	188	192	6	200	194	6	200	194
32	6	188	248	6	244	468	6	244	468
33	6	188	764	6	201	977	6	201	977
34	6	190	059	6	199	279	6	199	279
35	6	190	894	6	197	149	6	197	149
36	6	191	895	6	196	854	6	196	854
37	6	192	493	6	194	614	6	194	614
38	6	193	207	6	195	602	6	195	602
39	6	193	430	6	194	608	6	194	608
40	6	193	877	6	194	679	6	194	679
41	6	194	001	6	194	134	6	194	134
Opt.	6	194	083						

Experiment VII

Cycle Lower bound Upper bound Upper bound
 in cycle

1	4	955	266				
2	.	.	.	420	978	653	420 978 653
3	.	.	.	18	889	849	18 889 849
4	.	.	.	32	725	385	.
5	.	.	.	13	258	772	13 258 772
6	.	.	.	19	824	079	.
7	.	.	.	8	907	348	8 907 348
8	.	.	.	9	131	240	.
9	.	.	.	24	040	963	.
10	.	.	.	12	142	211	.
11	4	955	266	11	347	691	8 907 348
12	.	.	.	8	962	272	.
13	.	.	.	15	737	733	.
14	.	.	.	16	463	484	.
15	.	.	.	11	436	065	.
16	5	406	210	9	452	740	.
17	5	656	894	7	812	594	7 812 594
18	5	686	546	7	792	453	7 792 453
19	5	776	902	7	063	807	7 063 807
20	5	783	511	6	603	430	6 603 430
21	5	830	713	6	419	268	6 419 268
22	5	909	971	6	494	145	.
23	5	972	189	6	494	856	.
24	6	011	365	6	636	554	.
25	6	019	900	6	411	896	6 411 896
26	6	095	565	6	343	410	6 343 410
27	6	101	241	6	285	538	6 285 538
28	6	118	197	6	226	617	6 226 617
29	6	152	758	6	270	220	.
30	6	160	182				.
31	6	162	637	6	237	298	.
32	6	176	106	6	211	897	6 211 897
33	6	176	160	6	369	322	.
34	6	178	110	6	241	221	.
35	6	178	376	6	257	250	.
36	6	178	946	6	204	284	6 204 284
37	6	179	477	6	207	337	.
38	6	180	367	6	220	992	.
39	6	183	673	6	198	519	6 198 519
40	6	187	123	6	204	489	.
41	6	188	282	6	196	300	6 196 300
42	6	189	385	6	201	608	.
43	6	191	879	6	195	268	6 195 268
44	6	191	998	6	196	583	.
45	6	192	380	6	195	239	6 195 239
46	6	192	684	6	195	963	.
47	6	192	909	6	196	245	.
48	6	193	279	6	194	475	6 194 475
49	6	193	876	6	194	362	6 194 362
50	6	193	934	6	194	562	.
51	6	193	955	6	194	273	6 194 273
52	6	194	008	6	194	229	6 194 229
53	6	194	037	6	194	149	6 194 149
54	6	194	067	6	194	119	6 194 119
55	6	194	082	6	194	096	6 194 096
56	6	194	082	6	194	083	6 194 083
57	6	194	082	6	194	082	6 194 082
Opt.	6	194	082				

Experiment VII

Cycle Lower bound Upper bound
in cycle

51	6 155 479	6 217 055	6 217 055
52	6 156 325	6 246 694	6 217 055
53	6 159 980	6 233 950	6 207 439
54	6 162 433	6 207 439	6 207 439
55	6 165 530	6 219 665	6 207 439
56	6 166 276	6 255 475	6 207 439
57	6 167 304	6 203 156	6 203 156
58	6 174 252	6 204 795	6 203 156
59	6 174 285	6 232 446	6 203 156
60	6 174 658	6 216 015	6 203 156
61	6 179 234	6 206 756	6 203 156
62	6 179 734	6 216 195	6 203 156
63	6 184 267	6 203 395	6 203 156
64	6 184 400	6 204 963	6 203 156
65	6 184 812	6 222 934	6 203 156
66	6 185 156	6 208 211	6 203 156
67	6 187 253	6 196 791	6 196 791
68	6 187 304	6 203 073	6 203 156
69	6 188 305	6 200 165	6 203 156
70	6 188 313	6 198 050	6 203 156
71	6 188 832	6 197 506	6 203 156
72	6 188 832	6 212 718	6 203 156
73	6 189 363	6 206 746	6 203 156
74	6 189 611	6 205 447	6 203 156
75	6 190 138	6 201 875	6 203 156
76	6 190 377	6 196 474	6 196 474
77	6 190 402	6 196 963	6 196 474
78	6 191 317	6 195 509	6 195 509
79	6 191 822	6 202 651	6 195 509
80	6 191 852	6 201 116	6 195 509
81	6 191 966	6 199 467	6 195 509
82	6 192 466	6 198 687	6 195 509
83	6 192 596	6 205 027	6 195 509
84	6 192 596	6 200 974	6 195 509
85	6 192 606	6 199 687	6 195 509
86	6 192 668	6 198 395	6 195 509
87	6 192 698	6 197 971	6 195 509
88	6 192 825	6 196 784	6 195 509
89	6 192 840	6 197 349	6 195 509
90	6 192 995	6 197 053	6 195 509
91	6 193 053	6 195 434	6 195 434
92	6 193 082	6 197 490	6 195 434
93	6 193 095	6 195 789	6 195 434
94	6 193 334	6 195 478	6 195 434
95	6 193 354	6 195 307	6 195 434
96	6 193 407	6 196 583	6 195 434
97	6 193 514	6 195 140	6 195 434
98	6 193 559	6 196 744	6 195 434
99	6 193 579	6 194 652	6 195 434
100	6 193 588	6 195 310	6 195 434
101	6 193 623	6 195 043	6 195 434
102	6 193 623	6 195 121	6 195 434
103	6 193 671	6 194 319	6 195 434

1	4 955 266	217 047 863	217 047 863
2	6 955 266	17 830 734	17 830 734
3	6 955 266	16 319 236	16 319 236
4	6 955 266	17 830 734	17 830 734
5	6 955 266	16 319 236	16 319 236
6	6 955 266	17 830 734	17 830 734
7	6 955 266	16 319 236	16 319 236
8	6 955 266	17 830 734	17 830 734
9	6 955 266	16 319 236	16 319 236
10	6 955 266	17 830 734	17 830 734
11	6 955 266	16 319 236	16 319 236
12	6 955 266	17 830 734	17 830 734
13	6 955 266	16 319 236	16 319 236
14	6 955 266	17 830 734	17 830 734
15	6 955 266	16 319 236	16 319 236
16	6 955 266	17 830 734	17 830 734
17	6 955 266	16 319 236	16 319 236
18	6 955 266	17 830 734	17 830 734
19	6 955 266	16 319 236	16 319 236
20	6 955 266	17 830 734	17 830 734
21	6 955 266	16 319 236	16 319 236
22	6 955 266	17 830 734	17 830 734
23	6 955 266	16 319 236	16 319 236
24	6 955 266	17 830 734	17 830 734
25	6 955 266	16 319 236	16 319 236
26	6 955 266	17 830 734	17 830 734
27	6 955 266	16 319 236	16 319 236
28	6 955 266	17 830 734	17 830 734
29	6 955 266	16 319 236	16 319 236
30	6 955 266	17 830 734	17 830 734
31	6 955 266	16 319 236	16 319 236
32	6 955 266	17 830 734	17 830 734
33	6 955 266	16 319 236	16 319 236
34	6 955 266	17 830 734	17 830 734
35	6 955 266	16 319 236	16 319 236
36	6 955 266	17 830 734	17 830 734
37	6 955 266	16 319 236	16 319 236
38	6 955 266	17 830 734	17 830 734
39	6 955 266	16 319 236	16 319 236
40	6 955 266	17 830 734	17 830 734
41	6 955 266	16 319 236	16 319 236
42	6 955 266	17 830 734	17 830 734
43	6 955 266	16 319 236	16 319 236
44	6 955 266	17 830 734	17 830 734
45	6 955 266	16 319 236	16 319 236
46	6 955 266	17 830 734	17 830 734
47	6 955 266	16 319 236	16 319 236
48	6 955 266	17 830 734	17 830 734
49	6 955 266	16 319 236	16 319 236
50	6 955 266	17 830 734	17 830 734

Experiment X

Appendix 4: Values of dual variables in the basic experiments

The exhibits in this section show the development of the values of the dual variables that corresponded to the common constraints during the solution procedures of the basic experiments (only common constraints 1-7 considered, as the dual variables that referred to constraints 8-11 remained unchanged).

Cycle	Common constraint						
	1	2	3	4	5	6	7
1	0	0	0	0	7.90	0	0
2	0	0	532.5	0	7.90	0	0
3	296.6	0	506.0	0	7.90	0	0
4	275.4	206.8	542.7	0	7.90	0	0
5	120.0	110.8	520.5	9.96	7.90	-15.90	0
6	114.0	217.7	542.6	9.03	7.90	-14.34	0
7	132.2	89.0	678.2	1.88	7.90	-5.94	0
8	135.1	225.1	694.3	0.22	7.90	-5.38	0
9	265.0	2219.4	671.3	3.81	7.90	-5.35	0
10	572.0	123.3	914.3	18.0	0	20.0	0
11	131.8	236.3	619.0	18.0	0	-0.88	-50.0
12	330.6	249.3	818.7	18.0	0	1.44	-39.35
13	269.5	182.9	641.4	6.90	0	1.21	126.50
14	269.5	182.9	641.4	6.90	0	1.21	34.28
15	177.5	104.0	567.7	18.0	0	-7.96	59.45
16	179.6	143.6	568.1	18.0	0	-7.76	24.61
17	229.5	115.3	681.6	9.32	1.22	-3.39	57.63
18	216.4	114.6	658.8	11.92	3.05	-2.67	1.21
19	127.5	128.4	648.4	6.38	1.43	-6.85	-29.23
20	223.6	182.0	642.2	10.74	4.10	-3.36	14.83
21	131.8	115.4	604.7	10.62	4.92	-6.07	61.68
22	160.3	115.8	667.1	10.71	3.29	-5.86	56.98
23	160.4	116.0	591.5	10.66	3.23	-5.81	56.83
24	198.0	115.4	655.0	10.73	3.42	-4.42	53.94
25	229.9	124.5	603.5	10.64	4.43	-2.57	57.49
26	232.5	127.5	651.5	10.67	4.30	-2.53	54.08
27	205.5	115.4	704.6	10.53	3.20	-3.15	29.68
28	205.5	116.1	668.1	10.49	3.12	-3.10	28.71
29	219.0	123.1	678.7	11.07	4.01	-3.33	47.07
30	230.6	151.1	613.3	11.04	3.77	-2.97	58.00
31	228.7	150.8	629.4	11.00	3.86	-3.02	56.42
32	225.6	144.7	675.6	10.74	3.81	-3.18	51.36
33	211.4	139.0	641.3	10.94	4.11	-3.35	54.54
34	215.6	128.2	677.2	10.87	3.20	-3.47	58.05
35	218.7	127.1	656.3	10.84	3.04	-3.37	55.06
36	215.8	163.6	663.7	10.84	3.80	-3.23	56.40
37	216.8	153.5	666.8	10.87	3.65	-3.24	54.54
38	216.4	141.2	630.0	10.74	4.06	-3.18	53.06
39	220.0	163.1	653.3	10.83	3.78	-3.10	51.96
40	221.1	155.5	652.3	10.83	3.72	-3.12	52.03
41	214.3	162.7	662.3	10.81	3.75	-3.48	51.95
42	214.6	160.5	661.2	10.80	3.75	-3.41	52.16
Opt.	214.6	161.7	662.1	10.84	3.70	-3.39	51.85

Experiment I

Cycle	Common constraint						
	1	2	3	4	5	6	7
1	1664.7	0	0	0	0	0	0
2	0	0	466.9	0	0	0	0
3	0	0	544.6	0	0	0	0
4	206.5	387.4	509.7	0	0	0	0
5	88.5	246.1	619.7	0	0	0	0
6	143.2	276.9	668.9	0	0	-18.00	0
7	328.8	387.8	822.3	18.00	0	-11.34	0
8	714.8	386.5	804.0	18.00	0	-15.82	0
9	808.7	76.9	598.0	18.00	0	17.62	0
10	877.6	139.5	670.6	1.03	7.90	-5.25	126.50
11	117.8	191.6	586.9	18.00	0	-4.39	126.50
12	120.1	189.8	586.9	18.00	0	-4.37	-50.00
13	346.3	129.3	584.3	18.00	0	-2.20	60.85
14	337.0	97.0	676.6	18.00	0	-2.06	-50.00
15	332.1	173.1	672.0	18.00	0	-2.09	-50.00
16	311.2	179.2	669.4	0.15	0	-2.33	-38.34
17	268.4	428.8	668.2	0.15	0	-2.33	60.69
18	165.7	148.2	693.7	6.16	1.34	-4.32	-50.00
19	191.5	160.6	586.0	8.03	3.69	-2.59	-31.34
20	151.7	116.4	629.9	10.43	4.47	-5.50	24.22
21	267.0	153.0	693.1	9.03	2.67	-0.77	32.16
22	118.4	112.4	712.4	11.20	3.31	-4.07	63.44
23	207.8	120.2	685.9	11.22	3.43	-4.05	54.80
24	210.4	134.9	610.6	10.86	3.48	-4.00	47.50
25	212.7	279.4	676.7	10.48	4.17	-3.21	41.86
26	205.4	113.2	653.9	10.83	4.10	-3.20	61.13
27	204.4	119.0	653.7	10.88	4.07	-3.22	60.99
28	206.3	114.2	660.3	10.96	4.14	-3.18	51.36
29	201.4	159.5	652.4	11.21	3.73	-3.19	54.37
30	237.9	140.2	661.1	10.87	3.55	-2.54	57.07
31	201.5	126.7	635.4	10.59	4.03	-3.82	52.08
32	228.9	161.2	680.4	10.98	2.04	-2.82	48.51
33	216.7	176.9	638.9	10.88	3.32	-3.30	53.63
34	216.7	146.6	638.9	10.87	3.32	-3.30	53.63
35	203.1	115.5	663.6	10.87	3.61	-4.24	53.71
36	202.3	149.1	653.9	10.86	3.71	-3.84	55.51
37	210.9	163.9	663.4	10.77	3.79	-3.35	54.65
38	210.8	113.6	657.4	10.77	3.79	-3.36	54.48
39	210.8	117.4	657.1	10.83	3.72	-3.44	52.31
40	217.7	127.4	658.1	10.81	3.76	-3.55	49.48
41	211.7	133.4	646.5	10.89	3.62	-3.15	51.45
42	217.4	136.7	664.1	10.87	3.69	-3.08	51.75
43	215.0	133.4	659.3	10.84	3.69	-3.36	51.82
44	213.6	127.7	661.4	10.83	3.67	-3.39	51.75
Opt.	214.6	161.7	662.1	10.84	3.70	-3.39	51.85

Experiment III

Cycle	Common constraint						
	1	2	3	4	5	6	7
1	0	0	0	0	7.90	0	0
2	403.0	0	539.7	0	7.90	0	0
3	285.3	0	506.6	0	7.90	0	0
4	388.2	0	795.7	0	7.90	0	0
5	154.3	324.7	540.8	0	7.90	0	0
6	282.5	652.2	652.2	0	7.90	-18.00	0
7	199.2	308.3	756.5	0	7.90	-9.71	0
8	312.2	291.2	875.2	18.00	0	-5.92	0
9	848.0	355.5	1002.4	18.00	0	11.65	0
10	188.6	159.7	647.7	1.49	0	-4.49	126.50
11	159.0	156.5	643.8	2.63	0	-4.35	-17.71
12	232.3	122.7	590.5	18.00	7.68	-6.42	56.98
13	212.1	107.6	711.0	18.00	0.85	-3.87	-50.00
14	210.0	148.3	709.6	18.00	0.78	-3.98	-50.00
15	338.9	134.4	636.9	10.25	5.49	-0.77	-50.00
16	309.8	291.7	604.8	11.78	1.92	-2.11	-50.00
17	277.8	214.6	708.6	10.71	3.87	-0.59	68.78
18	284.3	219.8	718.8	10.82	3.88	-0.31	44.10
19	176.5	115.5	619.0	10.66	3.84	-5.84	57.60
20	176.8	115.5	618.7	10.54	3.75	-5.93	32.12
21	179.5	120.3	616.2	13.10	5.62	-3.19	54.63
22	205.6	117.3	644.6	11.73	2.74	-3.08	55.52
23	227.5	115.8	669.1	10.65	4.38	-3.18	46.27
24	219.8	135.7	663.8	10.79	0.87	-3.10	11.27
25	183.7	115.2	575.4	10.87	3.94	-3.99	32.11
26	183.7	115.2	627.5	10.87	3.94	-3.99	32.11
27	209.1	115.6	684.3	11.03	3.36	-3.92	53.16
28	234.1	124.8	652.8	11.20	3.51	-3.01	42.53
29	224.6	141.5	678.0	11.01	3.41	-2.83	49.41
30	219.2	119.3	603.2	10.80	3.63	-3.20	60.13
31	216.9	121.7	613.0	10.82	3.59	-3.39	57.17
32	214.2	122.1	625.6	10.89	3.71	-3.44	55.03
33	207.8	157.1	664.2	10.83	3.72	-3.46	58.82
34	204.3	155.8	649.3	10.87	3.72	-3.46	54.71
35	204.9	157.9	632.3	10.92	3.69	-3.99	51.82
36	218.0	144.0	639.9	10.89	4.03	-3.20	52.29
37	211.6	122.0	672.0	10.80	3.80	-3.61	53.42
38	209.1	123.6	648.4	10.82	3.80	-3.61	51.89
39	209.1	122.8	638.7	10.85	3.73	-3.40	52.11
40	214.7	129.4	639.5	10.81	3.63	-3.23	51.86
Opt.	214.6	161.7	662.1	10.84	3.70	-3.39	51.85

Experiment II

Cycle Common constraint

Cycle	1	2	3	4	5	6	7
1	249.3	162.5	684.4	11.19	5.62	-1.32	52.48
2	146.1	114.2	574.6	9.33	0	-7.45	56.21
3	179.7	162.5	651.6	10.82	0	-5.90	51.77
4	321.8	238.3	753.8	10.79	0	-4.17	23.26
5	224.2	198.5	682.5	10.64	3.43	-2.45	49.07
6	213.3	118.5	650.8	10.87	4.78	-4.39	53.28
7	214.1	128.8	662.2	10.81	4.13	-3.83	56.46
8	213.6	128.3	661.4	11.10	4.76	-3.20	50.40
9	214.3	129.0	662.1	10.87	3.96	-3.42	52.34
10	214.0	128.8	662.2	10.81	2.72	-3.88	52.21
11	214.3	129.0	661.9	10.81	3.43	-3.43	54.64
12	215.3	130.0	633.2	10.81	3.79	-3.03	54.11
13	214.9	129.6	639.0	10.81	3.73	-3.31	53.50
14	214.3	129.0	662.0	10.83	3.72	-3.43	52.12
15	214.4	129.1	662.1	10.85	3.70	-3.36	51.73
Opt.	214.6	161.7	662.1	10.84	3.70	-3.39	51.85

Experiment IV

Cycle Common constraint

Cycle	1	2	3	4	5	6	7
1	0	6800.1	0	0	7.90	-18.00	-50.00
2	2636.8	0	0	0	0	20.00	126.50
3	0	2246.8	362.2	0	0	20.00	126.50
4	0	0	566.1	0	0	20.00	126.50
5	585.4	585.4	509.0	0	0	20.00	126.50
6	607.3	159.2	800.8	0	0	20.00	126.50
7	730.5	1876.9	1001.8	0	0	20.00	126.50
8	178.7	244.8	594.6	0	0	-18.00	-30.44
9	186.0	259.6	563.7	18.00	7.90	-9.70	-50.00
10	193.0	274.8	563.7	0	0	0.46	-50.00
11	108.4	0	752.7	0	0	-5.65	-50.00
12	154.1	156.4	606.5	0	0	-8.84	91.79
13	90.7	163.5	601.8	1.93	0	-9.25	-23.53
14	163.8	183.2	644.8	0.42	0	-7.05	-22.25
15	141.4	119.6	582.2	19.00	5.26	-4.91	71.78
16	200.9	117.5	566.2	13.75	3.97	-6.02	45.67
17	152.2	109.8	636.4	11.39	7.90	-0.49	-8.50
18	185.0	131.2	631.4	11.53	0	-1.61	26.17
19	227.8	110.9	629.0	9.14	4.86	-2.08	59.61
20	243.9	129.7	634.1	9.21	2.68	-1.23	61.92
21	246.7	128.8	631.3	9.06	6.56	-0.90	26.32
22	261.1	181.1	652.5	11.23	0.15	-1.52	30.06
23	189.6	116.3	658.1	10.33	3.38	-4.14	46.00
24	208.6	116.7	609.9	10.98	4.97	-4.08	59.60
25	218.0	113.9	692.5	11.49	5.19	-3.32	53.75
26	234.2	121.6	653.0	10.80	5.14	-3.24	52.45
27	220.6	131.8	687.1	11.31	4.69	-2.73	57.36
28	213.6	129.8	622.8	10.97	4.25	-3.11	38.36
29	225.0	150.7	680.1	10.86	3.67	-3.32	58.22
30	224.7	133.7	655.4	10.86	3.68	-3.33	54.72
31	215.8	124.8	663.8	10.73	4.13	-3.21	47.22
32	220.1	140.0	665.7	10.90	3.35	-2.92	51.53
33	211.8	123.1	653.9	10.82	3.56	-3.46	51.31
34	214.9	128.2	662.4	10.82	3.68	-3.19	51.68
35	214.5	129.4	662.2	10.83	3.76	-3.48	52.04
36	214.1	128.6	661.8	10.84	3.64	-3.39	51.69
Opt.	214.6	161.7	662.1	10.84	3.70	-3.39	51.85

Experiment VI

Cycle Common constraint

Cycle	1	2	3	4	5	6	7
1	246.9	160.3	656.5	11.06	5.23	-1.87	52.48
2	197.4	161.7	660.2	10.82	0	-4.41	54.66
3	241.4	152.4	631.5	10.90	2.36	-1.47	-50.00
4	213.7	128.4	635.8	10.81	3.92	-3.86	42.27
5	214.1	128.9	664.9	10.81	2.87	-3.02	52.53
6	214.7	129.4	662.3	10.83	3.84	-3.44	56.46
7	214.7	129.4	662.3	10.83	3.50	-3.44	53.12
8	214.7	129.4	662.3	10.83	3.76	-3.44	50.91
9	215.0	129.7	662.4	10.87	3.60	-3.22	51.99
10	214.4	129.1	662.1	10.85	3.60	-3.36	51.71
11	214.3	129.1	662.0	10.84	3.71	-3.30	51.88
12	214.4	129.1	662.0	10.84	3.70	-3.40	51.85
13	214.6	129.3	662.1	10.84	3.70	-3.40	51.85
14	214.6	129.3	662.1	10.84	3.70	-3.40	51.85
Opt.	214.6	161.7	662.1	10.84	3.70	-3.39	51.85

Experiment V

Cycle	Common constraint									
	1	2	3	4	5	6	7	8	9	10
1	0	2534.8	0	0	0	-18.00	-50.00			
2	0	0	576.6	0	0	-18.00	-50.00			
3	582.9	0	520.0	0	0	-18.00	-50.00			
4	218.1	0	560.0	0	0	-12.56	-50.00			
5	450.4	387.4	570.1	0	0	-16.29	-50.00			
6	127.1	273.7	576.4	0	0	-17.46	-50.00			
7	444.2	279.3	576.9	18.00	0	-5.37	-50.00			
8	0	83.5	669.3	18.00	0	-1.63	-50.00			
9	0	83.5	537.3	18.00	0	-1.01	-50.00			
10	325.8	83.4	511.1	18.00	0	4.13	-50.00			
11	253.2	83.4	644.9	18.00	0	4.38	-50.00			
12	419.0	397.0	511.1	0	7.90	20.00	176.50			
13	309.4	359.4	896.7	0	7.90	20.00	176.50			
14	637.7	1987.6	911.6	18.00	7.90	20.00	176.50			
15	285.7	283.2	592.0	0	0	3.39	-50.00			
16	640.7	273.3	1076.6	0	0	8.39	-50.00			
17	129.0	131.2	607.5	18.00	7.90	-5.36	87.14			
18	139.0	131.2	607.5	18.00	7.90	-5.36	-24.06			
19	128.4	110.1	578.1	9.80	0	-7.71	59.39			
20	131.7	143.2	581.7	10.29	0	-7.41	59.61			
21	266.5	115.3	598.2	12.76	0	-5.66	0.49			
22	162.2	119.7	599.8	5.94	4.04	-5.53	8.38			
23	224.4	144.8	657.7	13.22	0	1.10	50.61			
24	287.1	195.1	656.9	11.43	0	0.95	63.71			
25	287.1	195.1	656.9	11.43	0	0.95	31.70			
26	284.7	190.5	686.0	11.71	3.77	-3.34	58.68			
27	248.2	178.8	694.1	10.46	3.24	-2.51	53.18			
28	194.6	117.7	663.0	12.00	3.98	-3.24	1.18			
29	234.4	115.0	658.1	10.67	1.80	-2.23	10.18			
30	213.6	115.4	592.3	10.57	4.07	-3.84	44.48			
31	208.2	115.2	653.5	10.48	3.98	-4.01	43.44			
32	216.9	128.2	727.5	10.50	4.12	-2.09	42.27			
33	214.5	125.8	706.1	10.51	4.11	-2.22	45.77			
34	199.7	135.2	604.0	10.75	3.50	-3.54	60.98			
35	201.3	140.6	672.1	10.78	3.38	-3.55	57.40			
36	217.3	139.6	609.9	10.93	3.44	-3.07	50.61			
37	220.9	116.1	691.4	11.19	2.94	-3.07	54.40			
38	220.7	164.1	678.1	10.90	4.05	-2.73	54.20			
39	218.2	151.9	641.8	10.98	3.43	-3.21	35.54			
40	205.8	144.3	654.2	10.92	3.93	-3.50	50.66			
41	232.1	125.4	646.2	10.95	3.73	-2.63	55.83			
42	222.2	147.6	646.1	10.79	3.58	-2.99	51.43			
43	219.9	163.5	627.5	10.80	3.82	-3.21	53.17			
44	219.3	150.6	638.8	10.82	3.88	-3.20	53.18			
45	209.5	160.2	646.4	10.81	3.62	-3.66	53.43			
46	212.6	157.8	638.7	10.77	3.48	-3.62	50.94			
47	212.8	162.8	665.2	10.84	3.68	-3.46	51.60			
48	213.1	157.9	658.8	10.88	3.76	-3.23	52.37			
49	216.5	156.9	662.1	10.85	3.59	-3.36	51.72			
50	219.2	162.5	657.6	10.82	3.78	-3.44	52.10			
Opt.	216.6	161.1	662.1	10.84	3.70	-3.39	51.85			

Experiment VII

Cycle	Common constraint									
	1	2	3	4	5	6	7	8	9	10
1	0	0	0	16.50	0	-18.00	0			
2	0	0	327.4	18.00	0	20.00	-50.00			
3	0	0	525.0	18.00	0	-9.36	-50.00			
4	76.7	0	520.4	18.00	0	-5.85	-50.00			
5	74.8	190.0	520.7	18.00	0	-5.63	-50.00			
6	193.3	193.3	507.8	18.00	0	-9.36	-50.00			
7	129.6	129.6	547.3	18.00	0	-6.24	-50.00			
8	128.8	94.5	543.3	0	7.90	-9.95	-50.00			
9	89.5	208.4	554.2	0	7.90	-10.69	-50.00			
10	151.2	155.0	549.0	0	2.69	-9.13	-50.00			
11	113.0	119.0	615.0	9.10	7.90	-4.38	-1.15			
12	154.6	119.0	607.5	9.64	7.90	-4.35	126.49			
13	156.0	119.1	614.1	9.59	7.90	-4.65	21.39			
14	184.5	99.0	631.4	9.01	0.23	-3.40	68.81			
15	186.0	142.1	628.8	9.02	0.85	-3.75	68.92			
16	181.0	115.5	634.6	8.87	0.31	-3.56	20.92			
17	284.5	247.7	625.8	11.27	3.91	-5.45	33.76			
18	418.7	128.0	721.7	11.83	4.07	-8.23	37.55			
19	350.8	243.7	679.5	11.57	4.58	-8.23	45.61			
20	326.5	256.8	689.5	10.01	4.80	-2.14	48.29			
21	220.1	117.5	645.4	12.00	4.53	-2.22	55.08			
22	191.2	136.0	670.7	10.57	3.58	-4.90	54.56			
23	191.1	115.8	616.2	10.98	2.79	-5.00	-18.64			
24	218.7	115.9	587.5	10.08	4.19	-3.47	-2.23			
25	218.1	116.9	579.4	10.09	4.14	-3.16	-7.85			
26	240.4	168.0	649.3	10.79	2.56	-2.60	51.13			
27	211.2	130.0	644.5	10.76	2.87	-3.17	52.11			
28	236.8	127.9	702.0	11.12	3.88	-3.05	42.87			
29	236.8	127.9	623.4	11.12	3.88	-3.05	42.87			
30	220.4	151.3	679.8	11.02	4.06	-2.98	50.37			
31	215.6	154.6	609.3	10.93	4.02	-3.31	60.62			
32	214.7	148.4	614.0	10.87	3.97	-3.46	56.27			
33	212.6	134.7	624.5	10.73	3.82	-3.79	51.28			
34	213.1	163.3	634.3	10.98	3.86	-3.43	55.53			
35	225.3	144.5	639.0	10.89	3.90	-2.93	54.38			
36	210.9	152.7	655.4	10.86	3.84	-3.52	52.04			
37	213.8	156.5	666.3	10.78	3.66	-3.45	54.47			
38	218.8	163.1	661.3	10.82	3.78	-3.13	51.95			
39	219.6	157.9	661.4	10.86	3.54	-3.11	51.59			
40	214.7	161.8	661.8	10.83	3.75	-3.42	51.89			
41	214.4	156.9	666.1	10.85	3.70	-3.34	51.89			
Opt.	214.6	161.7	662.1	10.84	3.70	-3.39	51.85			

Experiment VIII

Cycle Common constraint										
	1	2	3	4	5	6	7	8	9	10
51	215.0	201.8	658.5	10.33	3.95	-2.89	58.65			
52	223.4	194.4	656.3	10.30	4.23	-2.47	44.93			
53	210.4	125.0	652.5	11.05	4.77	-3.27	54.65			
54	195.4	176.0	608.4	10.90	2.88	-4.51	53.23			
55	209.7	126.3	598.4	10.51	3.48	-3.64	55.99			
56	202.3	137.2	675.7	10.56	3.38	-3.54	55.69			
57	212.4	147.0	627.8	10.94	3.67	-3.63	73.67			
58	223.9	114.4	679.3	10.61	4.11	-2.99	79.83			
59	226.3	117.4	680.8	10.54	4.08	-2.57	39.85			
60	229.8	132.5	687.7	10.25	3.70	-2.25	50.45			
61	242.7	159.1	647.8	11.07	3.80	-1.37	56.40			
62	222.5	145.9	644.2	10.86	3.92	-2.31	47.11			
63	221.9	160.1	648.0	10.73	4.40	-3.49	54.20			
64	208.4	144.0	672.1	10.28	2.97	-4.12	78.07			
65	238.2	100.9	671.0	10.80	3.33	-2.83	52.73			
66	226.1	144.4	670.3	10.83	3.45	-3.14	52.23			
67	194.8	187.6	660.3	10.85	4.00	-4.16	51.86			
68	195.0	142.5	658.3	10.84	3.99	-4.15	51.36			
69	208.0	153.6	645.8	10.88	4.10	-3.68	55.37			
70	205.1	132.7	645.8	10.87	4.09	-3.70	55.63			
71	203.3	114.9	630.9	10.46	3.79	-3.99	38.20			
72	203.3	117.6	630.9	10.46	3.79	-3.99	38.20			
73	195.6	163.6	648.7	10.85	2.83	-3.82	44.83			
74	200.1	123.6	644.7	10.87	2.97	-3.65	46.78			
75	214.2	162.5	628.0	10.90	3.60	-3.26	50.27			
76	214.1	112.4	636.1	10.86	3.65	-3.27	49.53			
77	213.7	132.7	640.5	10.86	3.69	-3.32	47.05			
78	214.1	131.7	638.4	10.73	4.04	-2.86	52.31			
79	217.0	162.4	630.0	10.71	3.94	-3.07	55.68			
80	221.5	164.7	633.7	10.71	4.04	-2.95	56.85			
81	221.1	147.8	645.5	10.72	4.06	-2.93	54.56			
82	210.4	115.1	648.3	10.81	3.39	-3.34	50.00			
83	209.3	115.1	648.5	10.76	3.44	-3.49	56.54			
84	203.3	116.4	648.5	10.76	3.44	-3.49	56.54			
85	210.5	162.8	643.1	10.77	3.67	-3.46	56.64			
86	212.8	116.5	651.8	10.79	3.54	-3.44	56.36			
87	215.6	136.0	639.7	10.82	3.60	-3.39	54.48			
88	209.6	139.8	665.9	10.72	3.58	-3.62	54.52			
89	210.9	153.8	652.1	10.72	3.60	-3.65	54.61			
90	211.2	162.4	649.5	10.85	3.83	-3.38	54.78			
91	215.5	127.8	645.6	10.76	3.28	-3.07	54.79			
92	220.3	124.8	641.9	10.84	3.66	-3.16	54.40			
93	220.4	141.1	639.6	10.83	3.58	-3.12	54.22			
94	211.5	121.9	657.3	10.79	3.85	-3.40	53.45			
95	206.8	157.5	650.3	10.76	3.73	-3.61	54.13			
96	212.4	144.3	657.4	10.76	3.73	-3.34	52.67			
97	220.4	140.4	655.0	10.79	3.91	-3.31	50.86			
98	216.3	126.8	663.0	10.84	3.78	-3.41	52.52			
99	215.4	142.6	645.1	10.85	4.01	-3.31	52.86			
100	219.8	129.1	669.4	10.87	3.72	-3.12	51.34			
101	213.5	128.6	666.9	10.87	3.67	-3.43	53.61			
102	215.9	137.5	664.0	10.84	3.70	-3.37	52.58			
103	219.8	159.2	659.4	10.80	3.75	-3.87	51.83			
91.	214.6	151.7	662.1	10.84	3.70	-3.38	51.85			

Cycle Common constraint										
	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	7.90	20.00	71.02			
2	0	1655.3	0	0	7.90	20.00	11.02			
3	0	0	530.2	0	7.90	20.00	11.02			
4	366.7	366.7	497.2	0	7.90	20.00	11.02			
5	353.7	0	544.9	0	7.90	20.00	11.02			
6	100.5	120.9	508.6	0	7.90	-18.00	126.50			
7	139.6	154.9	514.2	0	7.90	-12.12	126.50			
8	170.1	181.4	518.6	0	7.90	-7.53	126.50			
9	64.4	56.6	626.1	0	7.90	-18.00	126.50			
10	100.5	84.6	629.9	0	7.90	-13.20	126.50			
11	171.3	146.8	638.4	0	7.90	-2.60	126.50			
12	143.3	146.5	547.2	0	7.90	-10.40	12.18			
13	103.4	117.6	537.3	18.00	0	-8.05	71.11			
14	144.7	150.3	535.0	18.00	7.04	-5.51	39.40			
15	106.5	85.5	633.4	18.00	5.24	-6.25	41.30			
16	106.5	172.3	633.4	18.00	5.24	-6.25	41.30			
17	154.7	150.4	597.6	18.00	1.26	-7.61	76.05			
18	137.7	139.6	598.3	5.23	1.61	-5.15	31.31			
19	343.6	152.6	657.4	18.00	7.90	-5.30	11.27			
20	591.6	125.6	727.2	18.00	0	11.26	90.96			
21	612.7	223.2	734.9	18.00	0	12.06	-50.00			
22	612.7	569.3	734.9	18.00	0	12.06	-50.00			
23	809.2	372.6	7296.7	0	0	10.76	-50.00			
24	738.2	338.9	1178.5	0	0	6.14	113.54			
25	738.6	210.9	1179.2	0	0	6.15	-45.57			
26	291.3	276.3	618.9	18.00	0	-0.18	51.20			
27	203.3	179.1	735.3	18.00	0	-3.38	40.17			
28	212.6	171.8	695.2	15.99	0	-4.56	41.42			
29	213.7	165.6	687.4	11.49	7.90	1.11	59.18			
30	214.8	112.8	656.4	10.26	5.16	-0.95	-5.57			
31	274.4	131.4	667.3	10.69	6.77	-0.23	25.82			
32	257.4	167.7	627.2	9.33	4.58	-3.40	65.76			
33	253.1	220.5	640.1	10.12	5.65	-2.25	43.29			
34	214.7	117.2	658.9	10.36	2.62	-0.78	51.74			
35	264.7	220.5	687.7	11.71	3.66	1.04	56.78			
36	294.1	174.6	629.8	9.61	1.66	-2.88	47.87			
37	252.8	208.9	694.7	12.53	4.73	1.71	-50.00			
38	184.1	117.9	596.7	11.42	4.68	-4.10	35.74			
39	219.4	138.5	709.1	10.37	4.30	-1.83	39.00			
40	199.1	106.1	648.8	10.73	1.53	-3.36	54.12			
41	206.1	118.0	656.0	10.93	1.13	-2.72	43.96			
42	271.6	125.8	680.6	11.77	4.81	-2.23	58.15			
43	261.1	114.2	672.6	11.56	4.77	-2.76	38.65			
44	272.8	126.9	681.0	11.78	4.82	-2.20	36.80			
45	197.2	116.1	639.1	10.53	4.48	-5.24	56.45			
46	244.4	227.0	686.2	11.67	4.62	-2.39	44.77			
47	182.8	114.4	631.1	10.29	4.42	-5.82	0.56			
48	185.3	121.5	638.8	10.26	4.39	-5.60	57.10			
49	258.4	145.9	672.8	10.41	4.55	-1.35	57.10			
50	263.4	133.0	695.5	10.81	3.00	-2.85	52.18			

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER SOL 78-33	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) COMPUTATIONAL EXPERIMENTS ON LARGE-SCALE OPTIMIZATION WITH THE DECOMPOSITION PRINCIPLE		5. TYPE OF REPORT & PERIOD COVERED Technical Report
7. AUTHOR(s) Gerhard Schiefer		6. PERFORMING ORG. REPORT NUMBER SOL 78-33
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Operations Research -- SOL Stanford University Stanford, CA 94305		8. CONTRACT OR GRANT NUMBER(s) N00014-75-C-0267
11. CONTROLLING OFFICE NAME AND ADDRESS Operations Research Program -- ONR Department of the Navy 800 N. Quincy Street, Arlington, VA 22217		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NR-047-143
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		12. REPORT DATE December 1978
		13. NUMBER OF PAGES 43
		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
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17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
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19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Decomposition Large-Scale Optimization Linear Programming Decomposition Principle		
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